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A simple index of common ownership is not so simple

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1. Introduction

Last December, the Federal Trade Commission (FTC) held a hearing about whether common ownership reduces competition among firms[1]. Common ownership, as defined by the US antitrust agencies, is "the simultaneous ownership of stock in competing companies by a single investor where none of the stock holdings is large enough to give the owner control of any of these companies."[2] The event featured speeches from FTC Commissioner Noah Joshua Phillips, Securities and Exchange Commission (SEC) Commissioner Robert J. Jackson, as well as numerous industry participants, economists, legal scholars, and policymakers. The hearing covered three areas: the mechanism by which institutional investors with common ownership may have an impact on competition, the economic theory of the competitive harm from common ownership, and the empirical (econometric) evidence of competitive harm from common ownership. The hearing wrapped up with acknowledgements of the interesting and provocative nature of the early work in this field and calls for additional research in all three areas.

In this article, we take a step back from the debate about whether and how common ownership may result in competitive harm and focus on a commonly used index for measuring common ownership concentration to ask what it is measuring. In particular, we provide insight for practitioners into the frequently cited modified Herfindahl Hirschmann Index (MHHI) through numerical examples. The relationship between the Herfindahl Hirschmann Index (HHI) and MHHI might give the mistaken impression that common ownership concentration in an industry can be summarized by an index in a manner similar

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1 Hearings on Competition and Consumer Protection in the 21st Century, Hearing #8, New York University School of Law, an FTC-NYU School of Law Event, December 6, 2018, https://www.ftc.gov/news-events/events-calendar/ftc-hearing-8-competition-consumer-protection-21st-century.

2 Speech of Commissioner Phillips at the Hearings on Competition and Consumer Protection in the 21st Century, Hearing #8, Federal Trade Commission, Competition and Consumer Protection in the 21st Century Hearing #8, December 6, 2018, Hearing Transcript, hereafter Hearing #8 Transcript, p. 8 (available at https://www.ftc.gov/system/files/documents/public_events/1422929/ftc_hearings_session_8_transcript_12-6-18.pdf).

to industry concentration.[3] To the contrary, researchers in this area have cautioned that common ownership is complex and that the MHHI itself will reflect a number of other market features in addition to common ownership concentration, including the structure of non-common shareholders and firm market shares. As Daniel O'Brien noted at the FTC hearing, "the MHHI is a measure of concentration, not a measure of common ownership. So there are two issues here. One is that common ownership has multiple dimensions… and the MHHI has only a single dimension. So it's generally not possible to capture the impact of common ownership that way."[4] Our numerical examples elaborate on this.

The discussion is motivated by regulators' use of the MHHI and similar measures and the use of this measure in empirical research on the competitive effects of common ownership. For example, the European Commission has considered common ownership and the MHHI in recent merger analyses including Dow/Dupont. [5] Moreover, the MHHI and similar measures have figured prominently in the early empirical research on how common ownership affects market outcomes.[6] A better understanding of the MHHI should help inform the discussion of analyses that employ the MHHI.

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[3] This is not to say that the HHI and its implementation are without issues. However, as illustrated here, measuring common ownership concentration in an industry involves the ownership structure across firms and within firms and market structure, and so the MHHI will change with factors beyond common ownership and control.

[4] Federal Trade Commission, Competition and Consumer Protection in the 21st Century Hearing #8, December 6, 2018, Hearing Transcript (available at https://www.ftc.gov/system/files/documents/public_events/1422929/ftc_hearings_session_8_transcript_12-6-18.pdf).

[5] See European Commission Decision of 27 March 2017, Case M.7932 Dow/DuPont, pp. 380- 383 and Annex 5, pp.17-20. Although it considered the MHHI, the Commission stated that it ultimately did not rely on MHHI in its decision.

[6] See, for example, Azar, José, Martin C. Schmalz, and Isabel Tecu (2018). "Anticompetitive Effects of Common Ownership." *The Journal of Finance*, 73:4., hereafter Azar *et al*. Backus, Matthew, Christopher Conlon, and Michael Sinkinson, "The Common Ownership Hypothesis: Theory and Evidence, Brookings working paper, January 2019, hereafter Backus *et al*., discuss some of these papers and specifications, pp. 21-22.

2. MHHI, MHHID, and HHI

The MHHI has been described as "a generalization of the HHI that takes into account partial ownership."[7] It can be decomposed into two terms, the HHI and a second term, referred to as the MHHI delta (MHHID), to quantify the "additional concentration that arises because of common ownership."[8]

Our numerical illustrations are based on a simplified market and investor structure presented in O'Brien and Waehrer. This setting has three firms and a single common owner.[9] The common owner owns fractions β_1, β_2, and β_3 in firms 1, 2, and 3 respectively. We assume that control shares of firm j, γ_j, are equal to ownership shares so that $\gamma_1 = \beta_1$, $\gamma_2 = \beta_2$, and $\gamma_3 = \beta_3$.

In addition to the common owner, each firm j has I_j owners who own equal shares of the remaining portion of the firm but do not have a stake in any of the other two firms. We refer to these as non-common owners. These non-common owners also have equal control shares of the remaining firm control. So, for example, because the common owner owns β_1 of firm 1the non-common owners will cumulatively own the remainder or $(1- \beta_1)$. Each of the I_1 non-common owners will have an equal share of this so each will own $(1- \beta_1)/I_1$ of the firm. We assume that control shares are equal to ownership shares, so each non-common owner has control share of $(1-\gamma_1)/I_1$.

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7 O'Brien, Daniel and Keith Waehrer (2017). "The Competitive Effects of Common Ownership: We Know Less Than We Think." *Antitrust Law Journal* 81:3, hereafter O'Brien and Waehrer, p. 742.

Technically the MHHI is defined as

$$\text{MHHI} = \sum_j \sum_k s_j s_k \frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij} \beta_{ij}}$$

where j and k index firms and i indexes investors, s denotes firms' market shares, β_{ik} denotes investor i's share of firm k's equity, γ_{ij} denotes investor i's share of firm j's control.

The MHHI was generalized by Salop, Steven C. and Daniel P. O'Brien (2000). "Competitive Effects of Partial Ownership: Financial Interests and Corporate Control," *Antitrust Law Journal* 67, pp. 559-614; hereafter "Salop and O'Brien." It was based on the model developed to describe how cross-holdings and joint ventures affect competition in Bresnahan, Timothy and Steven C. Salop. (1986). "Quantifying the Competitive Effects of Production Joint Ventures." *International Journal of Industrial Organization*, 4(2), 155-175.

8 O'Brien and Waehrer, p. 743.

9 O'Brien and Waehrer, pp. 738-742.

In this setting, the MHHI is:

$$MHHI = s_1^2 + s_2^2 + s_3^2 + s_1s_2(C_{12}+C_{21}) + s_1s_3(C_{13}+C_{31}) + s_2s_3(C_{32}+C_{23})$$

where

$$C_{12} = \frac{\gamma_1\beta_2}{\gamma_1\beta_1 + \frac{(1 - \gamma_1)(1 - \beta_1)}{I_1}}$$

and the other C terms are defined similarly. The numerator of C_{12} reflects the control that the common owner has in firm 1 and the financial stake that it holds in firm 2, or the "'across-firms' concentration" due to common ownership.[10] The denominator of C_{12} "measures the 'within-firm' concentration of the joint ownership and control."[11]

The first three terms above are sum of squared market shares of firms 1, 2, and 3, or the HHI. The last three terms are the terms that reflect the control of firm j by owners of firm k, and, in sum, are the MHHID. When there is no common owner, so there is no investor with ownership in more than one competitor, the numerator of each C term will be zero and the last three terms will all be zero. Thus, when there are no common owners, the MHHI is just the HHI.[12]

As mentioned above, the MHHI reflects multiple aspects of firm and market structure – within-firm ownership and control concentration, across-firm concentration of ownership and control, and market shares. Our numerical calculations illustrate the complexity in comparing MHHI across markets and interpreting the differences as reflecting common ownership concentration because, in addition to the magnitude and structure of common ownership, the MHHI reflects the structure of the non-common owners and market structure. We start with numerical illustrations based on this simple setting where firms have equal market shares and expand it to consider two common investors and unequal market shares.

[10] Salop and O'Brien, p. 612.

[11] Salop and O'Brien, p. 612.

[12] Another way in which the MHHI generalizes the HHI is that when a common owner owns all of two firms, say firm 1 and firm 2, as in a merger of firms 1 and 2, then the MHHID is $2s_1s_2$ which is the change in the HHI from a merger of firms 1 and 2. See Salop and O'Brien, pp. 595-596, O'Brien and Waehrer, p. 743.

2.1 *As ownership shares of non-common owners become smaller, the common ownership incentive as measured by the MHHI increases*

Table 1 presents the MHHI for a market with one common investor who holds equal ownership shares in the three firms. The table shows how the MHHI changes as the size of the common ownership investment increases and as the number of non-common owners increases.

Table 1: MHHI for single common owner with equal shares in all three companies

	Common owner's share			
Non-common owners	**1%**	**5%**	**10%**	**20%**
	(a)	**(b)**	**(c)**	**(d)**
10	3,340	3,513	4,066	5,897
20	3,347	3,683	4,653	7,037
50	3,367	4,144	5,878	8,384
100	3,401	4,779	7,017	9,080
1,000	3,951	8,232	9,500	9,895
100,000	9,405	9,976	9,995	9,999

Notes: Calculations are based on a three-firm market with equally sized competitors (HHI = 3,333). Control share is proportional to ownership, hence equal to ownership share. The remaining, non-common, ownership is held by I shareholders with equal ownership and control shares (= (1- common holder share)/I).

Table 1 shows the MHHI for a common investor with 1%, 5%, 10%, and 20% ownership shares in each of the three firms. Column (a) presents the MHHI for a common investor with 1% share in each of the three firms as the number of non-common investors goes from 10 in the top row to 100,000 in the bottom row. The MHHI goes from 3,340 with 10 non-common investors to 9,405 for 100,000 investors. As the number of non-common owners increases, each has a smaller ownership share and control share. Therefore, control and ownership become more concentrated in the common owner as the remainder becomes dispersed among more non-common owners. Consequently, the MHHI increases, reflecting the changes in within-firm concentration of ownership and control.

Moving across a row, for a given number of non-common owners, the MHHI is increasing in the common owner's share of ownership and control. In the top row where each scenario has 10 non-common owners for each firm, as the common owner's share of the three firms increases from 1% to 5%, 10%, and 20%, the MHHI increase from 3,340 to 3,513, 4,066 and 5,897, respectively.

The incremental effect of going from 5% to 10% or from 10% to 20% common ownership and control in each firm depends on the within firm ownership structure, which in this example is determined by the number of non-common owners. As seen in the last row of Table 1, when there are 100,000 non-common owners the MHHI for a common owner with a

1% share in each firm is 9,405 while the MHHI for a common owner with a 20% share in each firm is 9,999. Common ownership stakes have larger impact on managers' decisions in the model underlying the MHHI when non-common owners are less concentrated.[13] As the number of non-common owners becomes very large, the common owner effect on the manager's decision becomes large regardless of the actual size of the common owner's share, and the common owner effect reflects the common owner's holding across firms. In these scenarios, because the common owner holds the same share in all three firms, the manager puts equal weight on the profit from the three companies. Thus, when the number of non-common owners becomes very high, the MHHI for a common owner with the same ownership share in each firm tends toward the monopoly level of 10,000.

2.2 Even when the incentives of common owners are perfectly aligned, a larger number of common owners will have a lower MHHI (and implied impact) than a smaller number of common owners with the same aggregate ownership.

Table 2 compares the MHHI for one and two common owners when the aggregate common ownership in each of the three firms is 20% and the common owners have the same incentives. In particular, we compare the MHHI for one common investor with 20% ownership and control in each firm in column (a), two common investors each with 10% ownership and control in each of the three firms in column (b), and two common owners-one with 5% and the other with 15% ownership and control in each of the three firms in column (c).

Table 2: MHHI Common Owners with Equal Shares and Aggregate Share of 20%

	Common owners' shares		
Non-common owners	**Owner 1: 20% Owner 2: 0%**	**Owner 1: 20% Owner 2: 0%**	**Owner 1: 15% Owner 2: 5%**
	(a)	**(b)**	**(c)**
10	5,897	4,921	5,206
20	7,037	5,897	6,257
50	8,384	7,398	7,743
100	9,080	8,384	8,641
1,000	9,895	9,793	9,834
100,000	9,999	9,998	9,998

Notes: Calculations are based on a three-firm market with equally sized competitors (HHI = 3,333). Control share is proportional to ownership, hence equal to ownership share. The remaining, non-common, ownership is held by I shareholders with equal ownership and control shares (= (1- common holder share)/I).

[13] See also the discussion in Backus *et al*, p. 15.

In all three scenarios and the common investors have the same incentives for profit maximization for each firm because they have equal shares in each of the three firms. The MHHI however, measures the extent of concentration of ownership and control and, with two common investors, ownership and control are generally less concentrated than with one common investor, all else equal. In addition, in column (b) with two investors with equal stakes of 10% in each firm ownership and control are less concentrated than in column (c) where one investor holds 15% in each firm and the other holds 5%. The results illustrate that the MHHI is affected by the number of common owners, even when common owners' interests are aligned and the aggregate common ownership is unchanged.

2.3 The similarity of common ownership structures as measured by the MHHI depends on the structure of within firm ownership and control.

Table 3 compares two scenarios from Table 2 above – one or two common investors with the same portfolio and aligned incentives – to two scenarios where there are two common investors with diverging incentives and with aggregate common ownership of 20%. In the first of the divergent scenarios the common investors hold stakes in all three firms: one investor holds 10%, 17%, and 3% of firms 1, 2, and 3 respectively while the other investor holds 10%, 3%, and 17% of the same firms. In the second of the divergent scenarios each common owner holds a stake in a firm the other does not. In column (d) one investor has shares of 10%, 20%, and 0% in firms 1, 2, and 3 while the other holds 10%, 0%, and 20% of the same firms.

Table 3: MHHI for common owners with aggregate share of 20%

	Common owners' shares			
Non-common owners	**Owner 1: 20% Owner 2: 0%**	**Owner 1: 10% Owner 2: 10%**	**Owner 1: 10,17,3% Owner 2: 10,3,17%**	**Owner 1: 10,20,0% Owner 2: 10,0,20%**
	(a)	**(b)**	**(c)**	**(d)**
10	5,897	4,921	4,578	4,290
20	7,037	5,897	5,274	4,805
50	8,384	7,398	6,264	5,530
100	9,080	8,384	6,871	5,975
1,000	9,895	9,793	7,691	6,580
100,000	9,999	9,998	7,806	6,666

Notes: Calculations are based on a three-firm market with equally sized competitors (HHI = 3,333). Control share is proportional to ownership, hence equal to ownership share. The remaining, non-common, ownership is held by I shareholders with equal ownership and control shares (= (1- common holder share)/I).

As shown above, when the number of non-common owners is small the MHHI with two perfectly aligned common investors (column (b)) are closer to those of the common

investors who are not perfectly aligned in columns (c) and (d) than to the MHHI of the single common investor in column (a). When the number of non-common investors is 10, the MHHI for two aligned common investors is 4,921 which is closer to that of the unaligned common investors in column (c) at 4,578, or in column (d) at 4,290 than that of the single common investor (by definition aligned) in column (a) at 5,897. When the number of non-common owners is larger, the MHHI in columns (a) and (b) for the single common owner and aligned common owners are closer than those of the common investors with divergent incentives in columns (c) and (d). With 1,000 non-common owners the MHHI for a single common investor is 9,895 (column (a)), for two aligned common investors it is 9,793 (column (b)), for two divergent investors with holdings in all three firms it is 7,691 (column (c)), and for two divergent common investors with no holdings in one of the three firms it is 6,580 in (column (d)).

2.4 Unlike the HHI, the MHHI is not bounded

The MHHI is not bounded by the monopoly level of HHI and can be above 10,000 when the number of non-common owners is large. Table 4 shows four scenarios where the MHHI is above 10,000 when the number of non-common owners is 1,000 or more.

Table 4: MHHI may be greater than 10,000

	Common owners' shares			
	One common owner		Two common owners	
Non-common owners	Owner 1: 5, 10, 20%	Owner 1: 3, 20,20%	Owner 1: 1.5, 10,10% Owner 2: 1.5, 10,10%	Owner 1: 12,6,4% Owner 2: 7,5,3%
	(a)	(b)	(c)	(d)
10	4,139	4,457	4,012	3,889
20	4,696	5,031	4,457	4,326
50	5,836	5,946	5,238	5,282
100	7,033	6,830	5,946	6,323
1,000	11,622	13,091	10,603	10,233
100,000	13,585	20,550	20,399	11,786

Notes: Calculations are based on a three-firm market. Control is proportional to ownership, control shares equal ownership shares. The remaining ownership is held by I non-common shareholders with equal ownership and control shares (= (1- common holder share)/I).

The first two scenarios in Table 4 have a single common owner: in column (a) the single common owner has a 5%, 10%, and 20% share in firms 1, 2, and 3 respectively; in column (b) the single owner has shares of 3%, 20%, and 20% in the three firms. The third and fourth scenarios involve two common owners: in column (c) each common investor holds the same ownership shares of 1.5%, 10%, and 10% in firms 1, 2, and 3; in column (d) the first common owner holds shares of 12%, 6%, and 4% while the second common owner holds shares of 7%, 5%, and 3% in the three firms. In each case the MHHI is above 10,000 with 1,000 non-common owners and it increases further as the number of non-common owners increases.[14]

2.5 The MHHI and MHHID are affected by changes in market shares

As is well known, MHHI and MHHID depend on market shares. Table 5 compares the MHHID and MHHI for a single common owner in the three firm industry with equal market shares – columns (a) through (e) – to those of a single common owner with the same ownership shares in the three firm industry with market shares of 10%, 25%, and 65% – columns (f) through (h). We consider two common ownership structures: the common ownership investment is 2%, 2%, and 3% for two scenarios – columns (b), (c), (g) and (h) – and ten times that, or 20%, 20%, and 30%, for the other two scenarios.

[14] This is not just a theoretical point as it appears to occur in practice. For example, Azar *et al* report MHHI up to 10,219, Azar *et al*, p. 1524). On a technical note, Backus *et al* include a brief discussion of the fact that the C terms are not bounded by 0 and 1 which leads to this result (Backus *et al* p. 4).

Table 5: MHHI and MHHID reflect market structure, common ownership, and non-common ownership

Non-common owners	Equal market shares					Unequal Market shares of 10, 25, 65%				
		Owner 1: 2, 2, 3%		Owner 1: 20, 20, 30%			Owner 1: 2, 2, 3%		Owner 1: 20, 20, 30%	
	HHI	MHHID	MHHI	MHHID	MHHI	HHI	MHHID	MHHI	MHHID	MHHI
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(I)	(k)
10	3,333	37	3,370	3,096	6,429	4,950	31	4,981	2,487	7,437
20	3,333	74	3,407	4,251	7,584	4,950	61	5,011	3,366	8,316
50	3,333	181	3,514	5,545	8,878	4,950	149	5,099	4,332	9,282
100	3,333	351	3,685	6,194	9,528	4,950	289	5,239	4,811	9,761
1,000	3,333	2,358	5,691	6,942	10,275	4,950	1,892	6,842	5,359	10,309
100,000	3,333	6,891	10,225	7,036	10,369	4,950	5,322	10,272	5,428	10,378

Notes: Calculations are based on a three-firm market with equally sized competitors (HHI = 3,333). Control share is proportional to ownership, hence equal to ownership share. The remaining, non-common, ownership is held by I shareholders with equal ownership and control shares (= (1- common holder share)/I).

As shown in Table 5, MHHI are generally higher with the more concentrated market structure and the same common ownership; however, MHHID are generally lower. With 20 non-common owners, the MHHI for the less concentrated industry with ownership shares of 2%, 2%, and 3% in column (c) is 3,370 while for the more concentrated industry it is 4,981 in column (d). The MHHID, which is sometimes referred to as the effect of common ownership concentration, is 74 for the less concentrated industry structure as reported in column (b) which is greater than the MHHID of 61 for the more concentrated industry in column (g). As in the other examples, as the number of non-common owners increases the MHHI for common investors with similar relative shares in firms tend to the same level. As the number of non-common owners increases the differences in MHHI across industry structure become smaller, and the differences between MHHID across industry structures become larger.

3. Discussion

As stated earlier, common ownership that has been the recent subject of debate can be described as "the simultaneous ownership of stock in competing companies by a single investor where none of the stock holdings is large enough to give the owner control of any of these companies."[15] Characterizing common ownership and its potential competitive impact will necessarily involve considering the magnitude and influence of the common owner relative to other shareholders and the potential interaction between market structure and the structure of common ownership and control concentration. Therefore it is perhaps not surprising that the MHHI, which might be thought of as a measure of common ownership concentration, reflects a number of elements in addition to common owners' holdings. As Daniel O'Brien stated, "common ownership has multiple dimensions… and the MHHI has only a single dimension. So it's generally not possible to capture the impact of common ownership in that way."[16]

This article provides some insight into what this complexity might mean for those who implement the MHHI in practice or compare MHHI across markets. Because the MHHI will reflect changes along these other dimensions, we suggest that before the MHHI can be used as a screen for industry conditions with potential competitive concerns, more analysis is needed to determine whether the MHHI will reflect the particular conditions for competitive harm that are of concern to regulators. Similarly, the use of MHHI or MHHID to capture cross-market variations in common ownership concentrations in statistical analyses warrants further scrutiny to ensure that it is measures what it is intended to measure.

[15] Speech of Commissioner Phillips, Hearing #8 Transcript, p. 8.

[16] Federal Trade Commission, Competition and Consumer Protection in the 21st Century Hearing #8, December 6, 2018, Hearing Transcript (available at https://www.ftc.gov/system/files/documents/public_events/1422929/ftc_hearings_session_8_transcript_12-6-18.pdf).